Quenched Supersymmetry

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We study the effects of quenching in Super-Yang-Mills theory. While supersymmetry is broken, the lagrangian acquires a new flavour $U(1 \mid 1)$ symmetry. The anomaly structure thus differs from the unquenched case. We derive the corresponding low-energy effective lagrangian. As a consequence, we predict the mass splitting expected in numerical simulations for particles belonging to the lowest-lying supermultiplet.

1. INTRODUCTION

If it finally turns out that nature can express herself in the language of supersymmetric theories, the understanding of the strongly coupled regime of the latter may be very important. From gluino condensation to many other issues in nowadays implementations of string theories, such understanding is pertinent. A first important step in this direction was made by Veneziano and Yankielowicz (VY) [1], when they derived the low energy effective lagrangian for pure $N = 1$ supersymmetric Yang-Mills theory (SYM).

The "a priori" primary tool for a direct study of strongly coupled field theories is lattice regularization. Its numerical implementation can be very time/resource consuming, and it is well known that the quenched approximation greatly reduces such costs, albeit at the price of the corresponding systematic error. Numerical results for the spectrum of the lowest-lying supermultiplet have been obtained in this approximation [2], and could be better justified by an analytical study of the effects of quenching in a SUSY theory. For recent unquenched results see [3].

In this talk we discuss the low energy effective lagrangian for quenched $N = 1$ supersymmetric Yang-Mills theory in the continuum [4], paralleling ref. [5].

2. SYM AND ITS SYMMETRIES

The SYM lagrangian is the simplest supersymmetric gauge theory and describes a vector supermultiplet with fermion and boson fields in the adjoint representation of the gauge group. No matter superfields (containing fermion and scalar fields in the fundamental representation) are present. The action is

$$S_{SYM} = \int dx \left\{ -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{i}{2} \bar{\lambda}^a \gamma^\mu D^{ab}_\mu \lambda^b \right\}, \quad (1)$$

where $a = 1, 2, 3$ is the adjoint index and D^{ab} is a covariant derivative. This lagrangian, beyond gauge symmetry and supersymmetry, is classically invariant under chiral $U(1)$ transformation (a gluino mass term would also spoil supersymmetry). There is no vector $U(1)$ symmetry, since gluinos are Majorana fermions. Moreover, it obeys naive scale invariance.

However, chiral $U(1)$ and scale invariance are broken by the chiral and trace anomalies, respectively:

$$\partial^\mu J_\mu = -c(g) F^a_{\mu\nu} \tilde{F}^{a\mu\nu}, \quad (2)$$

$$\Theta^\mu_\mu = c(g) F^a_{\mu\nu} F^{a\mu\nu}, \quad (3)$$

where $c(g) = \beta(g)/2g$ and $\beta(g)$ is the β-function of the theory. These two anomalies and the supersymmetric trace anomaly, $\gamma^\mu S_\mu = 2c(g)\sigma_{\mu\nu} F^a_{\mu\nu} \lambda^a$, form the anomaly supermultiplet.

Following ref. [6], VY obtained the low energy effective action, S_{VY} [1], by requiring that it reproduces the symmetries of the fundamental action, as well as its chiral, scale and supersymmetric anomalies:

$$S_{VY} = \int d^4x \left\{ \frac{9}{\alpha} (S^\dagger S)^{1/3}_D \right. \quad (4)$$

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$$+ \quad \left[\frac{1}{3}\left(S\log\frac{S}{\mu^3} - S\right)_F + h.c.\right]\Bigg\}\,,$$

where S is a chiral supermultiplet containing bound states of gluons and gluinos (SYM is believed to be confined such as QCD), and α and μ are free parameters. The component fields of the supermultiplet S aquire the common mass $m_S = \frac{1}{3}\alpha\mu$ due to supersymmetry.

Recently, several modifications to this lagrangian has been proposed [7]. We will study, however, the effect of quenching to the VY lagrangian (modified theories could be studied in the same approach).

3. qSYM AND ITS SYMMETRIES

Paralleling ref. [5], we implement quenching at the fundamental level by adding to the SYM lagrangian a ghost scalar field in the adjoint representation. The new action is:

$$\begin{aligned} S^q_{SYM} &= \int dx \left\{ -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \right. \qquad (5) \\ &+ \left. \frac{i}{2}\bar{\lambda}^a\gamma^\mu D^{ab}_\mu \lambda^b + \frac{i}{2}\bar{\eta}^a\gamma^\mu\gamma_5 D^{ab}_\mu \eta^b \right\}\,. \end{aligned}$$

Unlike in [5], the ghost kinetic operator is $\gamma_\mu\gamma_5 D^\mu$, as a vector bilinear acting on commuting Majorana fields gives zero. After the integration of the ghost degree of freedom in the functional integral, the ghost and fermion determinants cancel each other, implementing the quenched approximation. Introducing a set of generators σ^i $(i = 0, \ldots, 3)$ mixing fermionic and bosonic fields, the action can be written as:

$$S^q_{SYM} = \int dx \left\{ -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{i}{2}\bar{Q}^a\sigma^0\gamma^\mu_R D^{ab}_\mu Q^b \right\} ,(6)$$

where Q is the doublet $Q^a = (\lambda^a, \eta^a)$. σ^0 is the identity matrix, and $\sigma^{1,2,3}$ denote the Pauli matrices. The action is no longer supersymmetric, as new bosonic fields have been introduced with no fermionic counterparts. It is still gauge and classically scale invariant, though, and its $U(1)$ chiral symmetry is promoted to a Z_2 graded $U(1 \mid 1)$ chiral symmetry. The four currents associated to this symmetry are:

$$J^i_\mu = \bar{Q}^a\sigma^i\gamma^\mu_R Q^a\,. \qquad (7)$$

Only the J^3_μ current is anomalous, while J^0, J^1 and J^2 are not: the gluino and ghost loops give the same contribution to the anomaly (with opposite sign). This means that the chiral anomaly breaks $U(1 \mid 1)$ to graded $SU(1 \mid 1)$. The new anomalies of the theory are:

$$\begin{aligned} \partial^\mu J^3_\mu &= -2c(g)F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\,, \qquad (8) \\ \Theta^\mu_\mu &= \frac{\beta'(g)}{\beta(g)}c(g)F^a_{\mu\nu}F^{a\mu\nu}\,, \qquad (9) \end{aligned}$$

where β' is the β-function for the pure gauge theory (the fermion and ghost contributions to the β-function cancel).

The low-energy effective lagrangian should be invariant under $SU(1 \mid 1)$ and reproduce these new anomalies. These type of requirement fixed uniquely the supersymmetric low-energy theory, for the lowest dimension operators, at leading order in the momentum expansion [1]. Supersymmetry being absent in the present case, we briefly sketch below the method to derive the splitting in the mass spectrum induced by quenching.

Under the assumption that the $SU(1 \mid 1)$ chiral symmetry is spontaneously broken by a singlet scalar field invariant under full $U(1 \mid 1)$[2], we consider as the new fields of the effective theory

$$\begin{aligned} \chi &= \sigma_{\mu\nu}F^a_{\mu\nu}Q^a \quad \chi \to U_R\chi \qquad (10) \\ \phi^i &= \bar{Q}^a\sigma^i Q^a \quad \phi \to U_R\phi U^\dagger_L \qquad (11) \end{aligned}$$

where $\phi = \sigma^i\phi^i = \frac{1}{\sqrt{2}}\rho\Sigma$, with ρ a scalar field invariant under $U(1 \mid 1)$ and $\Sigma = e^{i\hat{\theta}}$ a pseudoscalar field in the exponential representation $(\hat{\theta} = \theta^i\sigma^i, i = 0, \ldots, 3)$. θ_3 is the $U(1 \mid 1)$ extension of the VY anomalous pseudoscalar, whereas the three *pseudoscalar* fields θ^i $(i = 0, 1, 2)$ are the Goldstone modes associated with the spontaneous symmetry breaking of $SU(1 \mid 1)$.

The low-energy effective lagrangian in terms of the component fields is:

$$\begin{aligned} \mathcal{L}_{qVY} &= \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}\left[\partial^\mu\theta_0\partial_\mu\theta_3 + \partial_\mu\theta^+\partial^\mu\theta^-\right] \\ &+ \frac{1}{2}\partial^\mu\theta_3\partial_\mu\theta_3 + \frac{i}{2}\bar{\chi}\gamma^\mu\partial_\mu\chi \end{aligned}$$

[2]This ansatz is suggested by a Coleman-Witten argument [8], and supported by numerical results for quenched simulations [2].

$$
\begin{aligned}
- &\quad \frac{c_0}{2}\mu\bar{\chi}\chi + 4\frac{\mu^2}{4c_2}\theta_3^2 - \frac{\mu^2}{4d_2}\left(\frac{\beta'}{\beta}\right)^2\sigma^2 \\
+ &\quad \text{interactions} + \mathcal{O}(1/N_c^2) \qquad (12)
\end{aligned}
$$

where we write only the mass terms of the potential (since at this stage we are interested in the mass splitting). c_0, μ, c_2 and d_2 are free parameters. To obtain this lagrangian several steps were performed:

- To impose that the most general lagrangian invariant under $U(1 \mid 1)$ is naively scale invariant, paralleling the analysis for QCD [9], and that the only $U(1 \mid 1)$ and scale breaking terms are logarithms, whose transformations give rise to the right anomalies, alike to [6].

- Only terms up to $(F^a_{\mu\nu}F^{a\mu\nu})^2$, $(F^a_{\mu\nu}\tilde{F}^{a\mu\nu})^2$ in the scalar potential are retained, based in $1/N_c$ arguments[3] .

- The algebraic equations of motion for the auxiliary fields $F^a_{\mu\nu}F^{a\mu\nu}, F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$ were used, in order to express the on-shell lagrangian in terms of the physical fields χ, ϕ only.

- To expand around true minimum, so as to obtain the mass term of the potential. The relevant fields are the scalar σ, the *pseudoscalars* θ_0, θ_3 and $\theta^{\pm}$ and the *fermions* χ_λ, χ_η. Recall that $\theta^{\pm}$ and χ_η obey fermionic and bosonic statistics, respectively.

The quadratic term and the kinetic term for the θ_3 field are interpreted as vertices, since in the quenched theory no resummation of the mass terms can be done [5].

We can perform the limit $\eta \to 0$ in order to recover the VY theory. In this limit,

$$
c_0 = \frac{1}{4c_2} = \frac{1}{4d_2} = \frac{1}{3}\alpha\mu\,. \qquad (13)
$$

When performing this limit the coefficients of the operators were assumed to be analytical in the ghost field dependence, but for the coefficients of the anomalous terms, which are not. This non-analiticity is responsible for the mass splitting of the VY supermultiplet.

The mass spectrum for the bound states of gluino and gluon fields is given by:

$$
\begin{aligned}
m_\sigma &= \frac{\beta'}{\beta}m_\chi\,, \\
m_\chi &= \frac{1}{3}\alpha\mu\,, \qquad (14)\\
m_\theta &= 2m_\chi\,,
\end{aligned}
$$

to be compared with $m_\chi = m_\sigma = m_\theta$ in the unquenched theory. The numerical results of [2] are in fair agreement with the predicted $\sigma - \chi$ mass splitting for SU(2), although an accurate measure of the quenched non-OZI contribution to the θ mass is still needed.

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[3] As usual, $F^a_{\mu\nu}F^{a\mu\nu}$ is an auxiliary field in the lowest order Lagrangian[6].